6.4 Work – Concept and Fact Sheet – Dr. Loveless

Concept:

For a constant force moved a certain distance:

$$\text{Work} = \text{Force} \cdot \text{Distance}$$

If force and/or distance are changing, then

$$\text{Work} = \int_a^b (FORCE)(DIST)$$

Your job is to find a pattern for force and distance.

How to approach problems…

- *Step 0*: Draw a picture of the *start* and *end* of the scenario
  - Is it a "Leaky Bucket" or "Stack of Books" problem?
- *Step 1*: Label! Including labeling a horizontal slice.
- *Step 2*: Find the patterns for FORCE and DIST for a given slice.
- *Step 3*: Integrate

Unit Notes

	Metric	**Imperial**
Mass	kg = kilograms	
acceleration due to gravity on earth	9.8 m/s^2	32 ft/s^2
Force	Newtons = N = kg·m/s^2	pounds = lbs
Distance	m = meters	ft = feet
Work	Joules = J = N·m	foot-pounds = ft-lbs

Conversion and abbreviations:

g = grams cm = centimeter
1000 g = 1kg 100 cm = 1 m

In = inch ft = feet mi = miles
12 in = 1 ft 5280 feet = 1 mi

Problem type 1 **("Leaky bucket"):**
One object gets lighter/heavier as lifted)

Idea: FORCE = weight of object at a particular height
DIST = "infinitesimal" distances moved along the way

For all these problems: Make sure you understand what x represents.

- FORCE = given function = $f(x)$
- DISTANCE = Δx
- WORK = $\int_a^b (Force)(Dist) = \int_a^b f(x)dx$

Problem type 2 **("Stack of Books")**
Lifting chain or pumping

Idea: FORCE = weight of a horizontal slice
DIST = distance moved by that slice

For chain: k = density of chain = force per distance

- FORCE = weight of slice = kΔx
- DIST = distance moved by slice (depends on labeling)
- WORK = $\int_a^b (Dist)(Force) = \int_a^b (Dist)kdx$

For pumping: k = density of liquid = weight per volume

- FORCE = k*volume = k*(area of horizontal slice)Δy
- DIST = dist moved by slice (depends on labeling, usually a-y)
- WORK = $\int_a^b (Dist)(Force) = \int_a^b (a-y)k(area\ of\ slice)dy$

Examples from old exams:

1. A rocket loses mass as it burns fuel. The mass of the rocket when it is x meters off the ground is given by

$$f(x) = \frac{40}{9.8} + \frac{20}{9.8}e^{-\frac{x}{2}} \text{ kg.}$$

How much work is done in the first 8 meters off the ground?

2. A 50 foot cable with density 4 lbs/ft is hanging over the side of a tall building. Find the total work done in lifting half the cable to the top of the building.

3. The figure shows a tank.
The front face of the tank has the shape of $f(x) = 4x^2$. Initially, there is fluid in the tank up to a height of 1 foot. The fluid weights 15 lbs/ft^3.
How much work is done to pump all the liquid to the top of the tank?