

6.4 Work – Concept and Fact Sheet – Dr. Loveless

Concept:

For a constant force moved a certain distance:

$$\text{Work} = \text{Force} \cdot \text{Distance}$$

If force and/or distance are changing, then

$$\text{Work} = \int_a^b (\text{FORCE})(\text{DIST})$$

Your job is to find a pattern for force and distance.

How to approach problems...

- *Step 0:* Draw a picture of the *start* and *end* of the scenario
 - Is it a “Leaky Bucket” or “Stack of Books” problem?
- *Step 1:* Label! Including labeling a horizontal slice.
- *Step 2:* Find the patterns for FORCE and DIST for a given slice.
- *Step 3:* Integrate

Problem type 1 (“Leaky bucket”):

One object gets lighter/heavier as lifted)

Idea: FORCE = weight of object at a particular height

DIST = “infinitesimal” distances moved along the way

For all these problems: Make sure you understand what x represents.

- FORCE = given function = $f(x)$
- DISTANCE = Δx
- WORK = $\int_a^b (\text{Force})(\text{Dist}) = \int_a^b f(x)dx$

Unit Notes

	Metric	Imperial
Mass	kg = kilograms	
acceleration due to gravity on earth	9.8 m/s ²	32 ft/s ²
Force	Newton = N = kg·m/s ²	pounds = lbs
Distance	m = meters	ft = feet
Work	Joules = J = N·m	foot-pounds = ft-lbs

Conversion and abbreviations:

g = grams cm = centimeter

1000 g = 1kg 100 cm = 1 m

in = inch ft = feet mi = miles

12 in = 1 ft 5280 feet = 1 mi

Problem type 2 (“Stack of Books”):

Lifting chain or pumping

Idea: FORCE = weight of a horizontal slice
 DIST = distance moved by that slice

For chain: k = density of chain = force per distance

- FORCE = weight of slice = $k\Delta x$
- DIST = distance moved by slice (depends on labeling)
- WORK = $\int_a^b (\text{Dist})(\text{Force}) = \int_a^b (\text{Dist})kdx$

For pumping: k = density of liquid = weight per volume

- FORCE = $k \cdot \text{volume} = k \cdot (\text{area of horizontal slice})\Delta y$
- DIST = dist moved by slice (depends on labeling, usually a-y)
- WORK = $\int_a^b (\text{Dist})(\text{Force}) = \int_a^b (a - y)k(\text{area of slice})dy$

Examples from old exams:

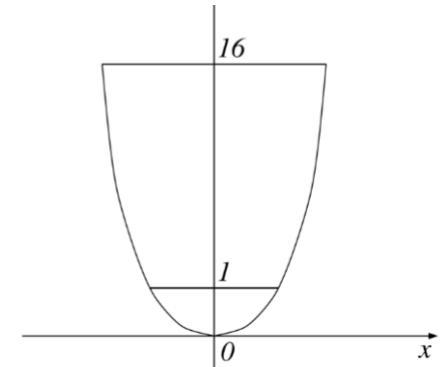
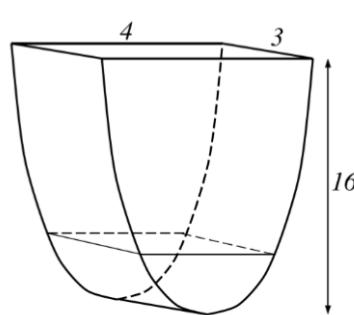
1. A rocket loses mass as it burns fuel. The mass of the rocket when it is x meters off the ground is given by

$$f(x) = \frac{40}{9.8} + \frac{20}{9.8} e^{-\frac{x}{2}} \text{ kg.}$$

How much work is done in the first 8 meters off the ground?

3. The figure shows a tank.

The front face of the tank has the shape of $f(x) = 4x^2$. Initially, there is fluid in the tank up to a height of 1 foot. The fluid weights 15 lbs/ft³. How much work is done to pump all the liquid to the top of the tank?



2. A 50 foot cable with density 4 lbs/ft is hanging over the side of a tall building. Find the total work done in lifting half the cable to the top of the building.